Gorter-Casimir two fluid model revisited and possible applications to superconductivity

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Abstract

In this paper, Gorter-Casimir (GC) two fluid model for low temperature normal superconductors is modified introducing phononic term along with electronic term in the normal phase free energy. The exponent corresponding to the normal phase fraction is changed from 1/2 to a general value *n* which can be different for different materials. *n* is a parameter which tunes how much portion of normal phase free energy will be reduced to form superconducting phase by condensation of normal electrons into super-electrons at some finite temperatures below superconducting transition temperature (T_C) and the electron-phonon interaction is the controlling factor which calibrate the values of n. This present model describes the idea of different jump ratios of specific heat of different materials at $T = T_C$, which GC model cannot predict. We have adopted a new concept of "Phase diagram" from which an idea of a new temperature T^* has been obtained. Modified GC model explains well the resistivity behavior near T_C. Moreover, for some high temperature superconductors along with the electronic and phononic contribution, a low temperature Schottky contribution is added to the free energy density. However, the contribution is negligible near T_C.

Keywords: Superconductors; Electron-phonon interaction; Two-fluid model.

1. Introduction

After the discovery of superconductivity of Hg at the liquid He temperature in 1911 by H. Kamerlingh Onnes, many research groups have tried to develop superconducting materials. Main motivation is to form superconductors which have superconducting transition temperature (T_C) near to room temperature. This is the temperature below which certain material becomes perfect conductors of electricity. Till now the highest achievable T_C is 203 K in H_2S at high pressure[1]. Many theoretical effort has been focused on the search for fundamental mechanism responsible for superconductivity [2-12] after the microscopic description of pairing mechanism by Bardeen-Cooper-Schrieffer (BCS) in 1957 [13]. The discussion of this microscopic pairing theory is beyond the scope of this article. Rather, we will concentrate on Gorter-Casimir (GC) two fluid model [14] for low temperature normal superconductors (LTS). It has been a well-known phenomenological model in order to describe various superconducting properties. This is the first phenomenological model before the theory of superconductivity introduced by Ginzburg and Landau in 1950. Phenomenological theories are some sort of theories which express analytically the results of observed phenomena without paying detailed attention of their fundamental microscopic significance [15]. Thus to describe some properties of superconductors e.g. the behaviour of critical field (H_C) variation with temperature, specific heat jump at T_C and the temperature variation of the depth to which magnetic field penetrates in a superconductor, two fluid model is a good analytical approach from the point of view of equilibrium thermodynamics [16, 17]. This model is very helpful for the novices to learn those superconducting properties quantitatively before understanding the microscopic theories. In 1934, Gorter and Casimir proposed the Helmholtz free energy density in superconducting phase as

$$F_s\,(x, T\,) = x^{\frac{1}{2}} f_n(\mathrm{T}) + (1 - x)\, f_s(\mathrm{T}) \qquad (1)$$

where x is the fraction of normal-electrons $(\frac{N_n}{N})$ and $1 - x$ is the fraction of superelectrons $(\frac{N_s}{N})$ in superconducting phase; where N, N_n and N_s are the number of total electrons, normal-electrons and super-electrons respectively. Here the normal-electrons and super-electrons are usually defined by electrons in normal metallic phase part and superconducting phase part, respectively. In normal metallic phase all the electrons are normal electrons. Below T_C, in the superconducting phase, it is thought that there is a co-existence of normal and super-electrons. Some normal electrons transforms to superelectrons. Here the "x" can be considered as an order parameter which characterizes the state of the system. $x = 1$ for $T > T_C$ and $x = 0$ for $T < T_C$. In the model they took the form of free energy density $f_n(\mathrm{T})$ and $f_s(\mathrm{T})$ as the following [14, 18].

$$f_n(\mathrm{T}) = -\frac{1}{2}\gamma T^2$$

$$f_s(\mathrm{T}) = -\beta$$

where β is the condensation energy of the normal electrons into super-electrons. They had considered only the normal specific heat per mole of conduction electrons $C_e = \gamma\mathrm{T}$. If the conduction electrons have effective masses m^* that differ from the free electron mass m, the conduction electron specific heat coefficient γ is given by, $\gamma = \frac{m^*}{m}\gamma_0$, where $\gamma_0 = 4.93R(\frac{1}{T_F})$, T_F is the Fermi temperature [19].

After minimizing the free energy of Eq. 1 w.r.t. x, one can get the temperature dependence of x. The temperature dependence of equilibrium x is denoted as $x = (\frac{T}{T_c})^4 \equiv x_{GC}$ [20]. Using this value the specific heat in superconducting phase is obtained as $C_s = 3\gamma T_c(\frac{T}{T_c})^3$ [20]. So, the ratio of the specific heat jump $(C_s - C_n)$ to the normal phase specific heat C_n at $T = T_C \left(\frac{(C_s - C_n)_{|T=T_c}}{C_{n|T=T_c}}\right)$ is found to be 2, while BCS theory [13] predicts that the value is 1.43. So, there is a little bit of discrepancy. A low temperature the GC model fails to reproduce the experimental specific heat data. But using the standard thermodynamic calculation the temperature dependence of HC can be obtained, which is good agreement with the experiment [21]. The temperature dependence of critical field is,

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \qquad (2)$$

where $H_c(T = 0) = \sqrt{\frac{1}{2}\frac{\gamma}{\mu_0}T_c^2}$ [16,17]; Unit of H_c is A/m. Moreover the variation of London penetration depth with temperature can be calculated as $\lambda_L(T) = \lambda_L[1 - \left(\frac{T}{T_c}\right)^4]^{-\frac{1}{2}}$ which is good agreement with J. G. Daunt et al. result [22]. Surprisingly, Gorter and Casimir have disregarded the lattice specific heat in their approach whereas on the other hand, according to Fröhlich [23] the electron-phonon interaction can provide attractive potential to bind electrons. Latter Bardeen, Cooper and Schrieffer (BCS) [13] used the idea of Fröhlich on the attractive interaction to give the microscopic theory of electron-pairing in superconductors. So, to modify the superconducting-state specific heat (C_s), it is necessary to consider the form of lattice specific heat $C_{ph} = AT^3$ (according to Debye model)‡ and use it in the form of total specific heat (C_n) of normal phase along with $C_e = \gamma$T. Thus, the specific heat of normal electrons has to be C_n=C_e+ C_{ph}. For most of the superconductors, the specific heat decreases monotonically to low temperatures. Interestingly, for some high temperature superconductors (HTS), a specific heat upturn at very low temperatures indicates a Schottky-type contribution in superconducting-state heat capacity. However, the Schottky contribution decreases rapidly with increasing temperature. The Schottky term has the form $C_{Schottky} \propto T^{-2}$. Consequently, the free energy in superconducting phase has to be modified.

Furthermore in GC model, the supposition of the exponent of x was taken on ad-hoc basis to compare the experimental results. Afterwards a specific example of a two fluidmodel that is different from the GC model had been developed by Koppe [24], discussed by Bender and Gorter [18]. Latter Lewis used the general form of two fluid model to calculate the specific heat in superconducting phase, temperature dependence of critical magnetic field and described “Energy-Gap” model and its consequences [25]. Vendik et. al. used an enhanced two-fluid model based on Gorter and Casimir idea using a general exponent of normal phase fraction to describe microwave properties of HTS [26]. So, instead of $\frac{1}{2}$, a general exponent (n) of x can be taken in order to investigate the contribution of the exponent in the various properties of superconductivity. The motivation of the present article is to modify the free energy of normal and superconducting phase part and to construct phase diagram of the superconducting phase transformation from normal metallic phase for some LTS and HTS. Furthermore, the present paper aims to calculate the specific heat in the superconducting phase and estimate the jump of specific heat at T_C with the modified free energy. Moreover, an investigation regarding the temperature dependence of critical magnetic field has been performed.

2. Mathematical formulation

2.1 Model for low temperature normal superconductor

Considering conduction electron specific heat and lattice specific heat, the specific heat of normal phase part is of the form

$$C_n = C_e + C_{ph}$$

$$= \gamma T + AT^3 \qquad (3)$$

where $A = 234R(\frac{1}{\theta_D})^3$ where θ_D is the Debye temperature [19]. So, the free energy density in the normal phase can be written as

$$f_n(\mathrm{T}) = -\frac{1}{2}\gamma T^2 - \frac{1}{12}AT^4 \qquad (4)$$

Using Eq. 4, the general expression of Helmholtz free energy density in superconducting phase can be written as,

$$F_s\,(x,T\,) = x^n[-\frac{1}{2}\gamma T^2 - \frac{1}{12}AT^4] + (1-x)\,(-\beta) \qquad (5)$$

This is named as Modified Gorter-Casimir (MGC) two fluid model. Minimizing the free energy of Eq. 5 w.r.t x, the temperature dependence of x is obtained as,

$$x = \left[\frac{n(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4)}{\beta}\right]^{\frac{1}{1-n}} \qquad (6)$$

where < 1 . At T = T_C , x=1; we derive the value of β and replacing β in equation 6 we get,

$$x = \left[\frac{(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4)}{(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4)}\right]^{\frac{1}{1-n}} = x_{MGC} \qquad (7)$$

Using the form of x from Eq. 7 in Eq. 5, the Helmholtz free energy density in superconducting phase can be written as,

$$F_s\,(x, T\,) = -(1-n)\left[\frac{\left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)^{\frac{1}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right] - n\,(\tfrac{1}{2}\gamma T_c^2 + \tfrac{1}{12}AT_c^4) \qquad (8)$$

Now using Eq. 8, the specific heat in superconducting phase (C_s) can be obtained by the relation $C_s = -T\,\frac{d^2F_s}{dT^2}$,

$$C_s = \frac{n}{1-n}\gamma^2 T^3\left[\frac{\left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)^{\frac{2n-1}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right]\left[1 + \frac{1}{3}\frac{A}{\gamma}T^2\right]^2 + \gamma T\left[\frac{\left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)^{\frac{n}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right]\left[1 + \frac{A}{\gamma}T^2\right] \qquad (9)$$

Subtracting Eq. 3 from Eq. 9 and dividing by Eq. 3, $\left(\frac{(C_s - C_n)_{|\mathrm{T=T}_c}}{C_{n|\mathrm{T=T}_c}}\right)$ can be obtained as,

$$\frac{(C_s - C_n)_{|\mathrm{T=TC}}}{C_{n|\mathrm{T=TC}}} = \frac{2n}{1-n}\frac{\left[1 + \frac{1A}{3\gamma}T_c^2\right]^2}{\left[1 + \frac{1A}{6\gamma}T_c^2\right]\left[1 + \frac{A}{\gamma}T_c^2\right]} \qquad (10)$$

Moreover, using standard thermodynamics, the condensation energy can be written as,

$$F_s\,(T\,) - F_n\,(T\,) = -\frac{\mu_0}{2}H_c^2(T) \qquad (11)$$

H_c is the critical filed at which the normal phase and the superconducting phase are in thermodynamic equilibrium. The concept of this critical magnetic field was first found out by Meissner and Ochsenfeld in 1933. The Meissner-Ochsenfeld effect is the observation that upon cooling the superconducting material below T_c the external applied magnetic field is expelled. Thus at $T < T_c$, the superconductor behaves as a perfect diamagnet. H_c is a limiting external applied magnetic field above which superconductivity is destroyed even at $T < T_c$. The temperature dependence of H_c can be obtained using the form of Eq. 8 and 4 in Eq. 11.

$$\mathrm{H}_c(T) = \sqrt{\frac{2}{\mu_0}}\left[(1-n)\frac{\left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)^{\frac{1}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}} + n\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right) - \left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)\right]^{\frac{1}{2}} \qquad (12)$$

For GC model, n = 1/2. Accordingly, the normal electron phase fraction (x), superconducting specific heat (C_s), specific heat jump ratio at $T_\mathrm{C}\left(\frac{(C_s - C_n)_{|T=T_c}}{C_{n|T=T_c}}\right)$ and the temperature dependence of H_c are respectively of the following form.

$$x = \left[\frac{(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4)}{(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4)}\right]^2 = \left(\frac{T}{T_c}\right)^4\left[\frac{1 + \frac{1A}{6\gamma}T^2}{1 + \frac{1A}{6\gamma}T_c^2}\right]^2 \qquad (13)$$

$$\frac{(C_s - C_n)_{|\mathrm{T=TC}}}{C_{n|\mathrm{T=TC}}} = 2\frac{\left[1+\frac{1A}{3\gamma}T_c^2\right]^2}{\left[1+\frac{1A}{6\gamma}T_c^2\right]\left[1+\frac{A}{\gamma}T_c^2\right]} \quad (14)$$

and

$$\mathrm{H}_c(T) = \sqrt{\frac{2}{\mu_0}}\left[\frac{1}{2}\frac{\left(\frac{1}{2}\gamma T^2+\frac{1}{12}AT^4\right)^2}{\left(\frac{1}{2}\gamma T_c^2+\frac{1}{12}AT_c^4\right)} + \frac{1}{2}\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right) - \left(\frac{1}{2}\gamma T^2 + \frac{1}{12}AT^4\right)\right]^{\frac{1}{2}} \quad (15)$$

2.2 Model for high temperature superconductor

Although for HTS the normal phase specific heat is similar to the form of Eq. 3, the superconducting specific heat of Eq. 9 should have low temperature Schottky correction. Thus superconducting specific heat is gien by

$$C_s = \frac{n}{1-n}\gamma^2 T^3\left[\frac{\left(\frac{1}{2}\gamma T^2+\frac{1}{12}AT^4\right)^{\frac{2n-1}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2+\frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right]\left[1+\frac{1}{3}\frac{A}{\gamma}T^2\right]^2 + \gamma T\left[\frac{\left(\frac{1}{2}\gamma T^2+\frac{1}{12}AT^4\right)^{\frac{n}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2+\frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right]\left[1+\frac{A}{\gamma}T^2\right] + \alpha T^{-2} \quad (16)$$

The Helmholtz free energy density in superconducting phase for HTS is of the form as shown in Eq. 17.

$$F_s\,(\,T\,) = -(1-n)\left[\frac{\left(\frac{1}{2}\gamma T^2+\frac{1}{12}AT^4\right)^{\frac{1}{1-n}}}{\left(\frac{1}{2}\gamma T_c^2+\frac{1}{12}AT_c^4\right)^{\frac{n}{1-n}}}\right] - n\,\left(\frac{1}{2}\gamma T_c^2 + \frac{1}{12}AT_c^4\right) - \frac{1}{2}\,\alpha T^{-1} \quad (17)$$

The specific heat jump ratio at T_c is of the form of Eq. 10. As the Schottky contribution decreases with increasing temperature, $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$ has negligible contribution of Schottky anomaly. Furthermore, the temperature dependence of H_c can be considered similar to Eq. 12 because in presence of high external field the Schottky contribution becomes suppressed at low temperature which was confirmed by Woodfield *et. al*. result [27].

3. Results and Discussions

For most LTS the transition temperature T_c is sufficiently below θ_D, so that the electronic term in the specific heat is appreciable in magnitude and sometimes dominates. This is not the case for HTS [19]. Using measured values of γ and A, Poole *et al.* [28] showed in their early work that, $AT_c^2 \gg \gamma$ for

Table 1. Different parameters values of Cd, Al, Sn (white), Pb, Nb, YBCO and TBCCO [19, 29, 30]

Material	T_c (K)	θ_D (K)	γ	A (mJ/mole-K^4)	$\frac{(C_s - C_n)_{\|T=T_c}}{C_{n\|T=T_c}}$
Cd	0.55	252	0.67	0.122	0.122
Al	1.2	425	1.36	0.026	0.026
Sn	3.72	196	1.78	0.258	0.258
Pb	7.19	102	3.14	1.833	1.833
Nb	9.26	277	7.66	0.092	0.092
YBCO	92	410	4	0.035	0.035
TBCCO	110	260	~ 6	0.111	0.111

$(La_{0.9}Sr_{0.1})_2CuO_{4-\delta}$ and $YBa_2Cu_3O_{7-\delta}$ .So for oxide superconductors the vibrational term dominates at T_c. Although, we want to investigate how much the phononic term contributes along with electronic term for some LTS like Cd, Al, Sn (white), Pb, Nb and some HTS like $YBa_2Cu_3O_7$ (YBCO), $Tl_2Ba_2CaCu_2O_8$ (TBCCO) etc. In Table 1, γ, θ_D, T_c , A and $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$ of these materials are tabulated. To understand how the value of specific heat jump ratio at T_c is modified over the original GC model due to inclusion of C_{ph}, values of A, γ and T_c of different materials are incorporated in the Eq. 14. Calculated and experimental $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$ values are listed in Table 2.

Table 2. Calculated and experimental ratio of specific heat jump and normal phase specific heat at $T = T_c$, for Cd, Al, Sn (white), Pb, Nb, YBCO and TBCCO

Material	Calculated ratio	Experimental value [19]
Cd	1.95	1.36
Al	1.97	1.45
Sn	1.39	1.60
Pb	1.30	2.71
Nb	1.52	1.93
YBCO	1.32	3.6
TBCCO	1.33	5.8

From Table 2, it is observed that the calculated values of $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$ are very much deviated from experimental values. Although, the calculated values for Sn, Pb, Nb, YBCO and TBCCO are very close to that of BCS prediction. This problem of deviation of calculated ratios from experimental values can be resolved by calibrating the exponent n of "x". n is a parameter which tunes how much portion of normal phase free energy will be reduced to form superconducting phase by condensation of normal electrons into super-electrons at some finite temperatures below T_c. Considering the experimental values of $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$ and using Eq. 10, we can get the values of n for different materials. In Table 3 the values of n are listed for different materials.

Table 3. Values of n for Cd, Al, Sn (white), Pb, Nb, YBCO and TBCCO corresponding to experimental values of $\frac{(C_s-C_n)_{|T=T_c}}{C_{n|T=T_c}}$

Material	Experimental ratio	n
Cd	1.36	0.411
Al	1.45	0.423
Sn	1.60	0.535
Pb	2.71	0.675
Nb	1.93	0.560
YBCO	3.6	0.732
TBCCO	5.8	0.814

The possible origin of increased specific heat jump at T_c in Pb, Nb, YBCO and TBCCO is increasing phonon contribution in superconducting phase below T_c . Poole *et.al.* [28] showed that $AT_c^2 \gg \gamma$ for some HTS, e.g. $(La_{0.9}Sr_{0.1})_2CuO_{4-\delta}$ and $YBa_2Cu_3O_{7-\delta}$ etc. Consequently, electron-phonon interaction increases which gives rise to more normal electrons condensed to super-electrons near T_c which is reflected in the value of "*n*". To understand this behavior the temperature variation of "*x*" is plotted using Eq. 7. In Fig.1,

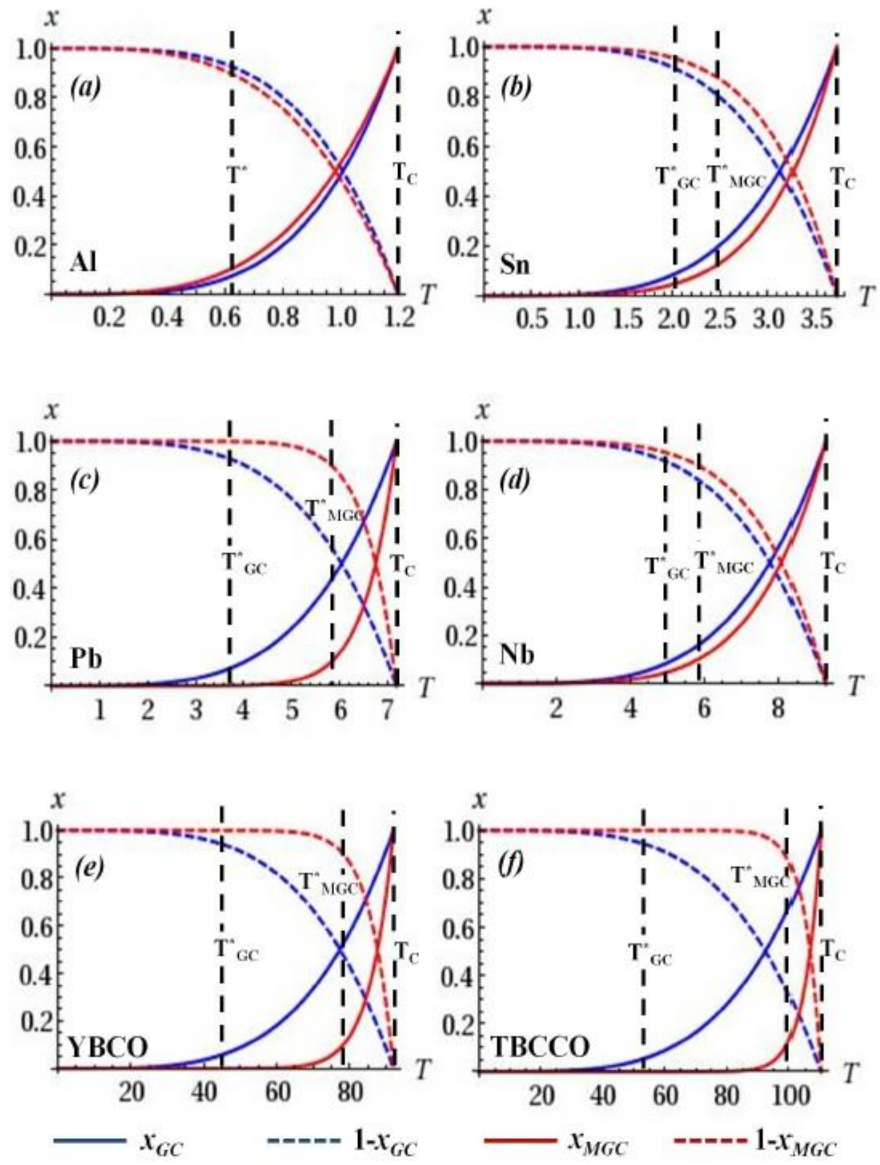

FIG 1. Phase diagram of Al, Sn, Pb, Nb, YBCO and TBCCO

the temperature variation of x_{GC}; $1 - x_{GC}$, x_{MGC} and $1 - x_{MGC}$ of the above materials are depicted. This can be named as "Phase diagram" of superconductors. At T_c all electrons are normal electrons. As the temperature is decreased normal electrons starts to form super-electrons. Below T^* temperature, all electrons are condensed to super-electrons. For Al, T^* for GC model (T_{GC}^*) and MGC model (T_{MGC}^*) are almost same. For Sn, Pb, Nb, YBCO and TBCCO T_{MGC}^* is greater than T_{GC}^* and $T_{MGC}^* - T_{GC}^*$ increases more in YBCO and TBCCO high temperature superconductors. This is because phonon contribution increases from LTS to HTS and more normal electrons are transformed to super-electrons. At T_c, the resistivity starts to fall rapidly due to start of formation of super-electrons and below certain temperature resistivity becomes almost zero (Resistance ~ 10^{-5} ohm) because the condensation to super-electrons completes below this temperature. This later temperature can be related to the T^* temperature. Here the rapid change in electron phase fraction below T_c to T^* adopted in MGC model can be attributed to sharp decrease in resistivity. MGC model explains well the resistivity behavior near T_c.

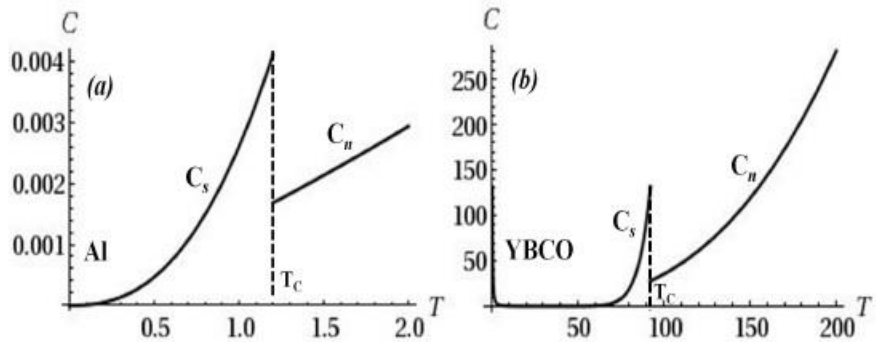

FIG 2. Specific heat of Al and YBCO; C is in J/mol-K.

In Fig.2, the temperature dependence of specific heat obtained by MGC model in normal and superconducting phase of LTS Al and HTS YBCO are shown. There is specific heat jump at TC for both the systems. Interestingly, for YBCO there is a specific heat upturn at very low temperature which is the effect of Schottky anomaly.

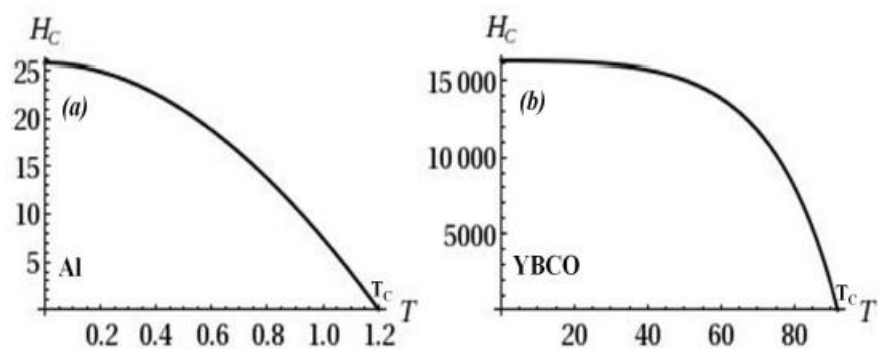

FIG 3. Temperature variation of HC for Al and YBCO

Beside perfect conductivity perfect diamagnetism is the essential hallmark of superconductivity. The later property of any superconductor is characterized by the critical field H_c. In Fig.3 the temperature variation of HC is shown for Al and YBCO. The variation is quite well agreement with experiment [21]. Here the LTS Al is type I superconductor. It has only one type of H_c which is termed in literature as H_{c1}. Here the magnetic state is based on Meissner phase of diamagnetism where there is no vortex state. It is not the case for type II superconductor where there exists an upper critical field (H_{c2}) other than H_{c1}. Variation of $\mathrm{H}_{c2}$with temperature is similar to H_{c1}. Here Meisnner magnetic state is obtained at low temperatures with magnetic field $H_{ext} < H_{c1}$. At higher temperatures mixed state (vortex lattice) is formed in the range $H_{c1} < H_{ext} < H_{c2}$. YBCO is an example of high temperature type II superconductor. Although here only the temperature variation of H_{c1} is depicted. To handle the vortex state we need microscopic theory which is beyond the scope of this simple model.

4. Conclusion

In the summary, we would like to emphasize the basic results of this article more precisely. Gorter-Casimir two fluid model is modified introducing phononic term along with electronic term in the normal phase free energy. The specific heat jump ratio of different materials obtained by this modified model are very close to the value (1.43) predicted by BCS. Although, the calculated values are very much different from experimental values. So, only inclusion of phononic term is not sufficient to predict experimental values of $\frac{(C_s - C_n)_{|T=T_c}}{C_{n|T=T_c}}$. To resolve this problem a general exponent of normal phase fraction has been considered in the expression of Helmholtz free energy density of superconducting phase. It is observed that, the value of n has to be very different from $\frac{1}{2}$ in order to match the jump in the theoretical value of the heat capacity at T_c with the experimental value for different materials. This result is completely new in the context of earlier results, obtained by Gorter and Casimir where they have considered the exponent as constant. n which tunes how much portion of normal electrons condensed to super-electrons, increases from LTS to HTS because of increased specific heat jump ratio. This is because electron-phonon interaction increases from LTS to HTS. This gives rise to more condensation of normal electrons to super-electrons near T_c. Moreover, from “Phase diagram” we have got an idea of a new temperature T^* and explained the resistivity behavior near T_c. At T_c the resistivity starts to fall rapidly due to start of formation of super-electrons and below T^* resistivity becomes almost zero because of completion of condensation of normal electrons to super-electrons. The rapid change in electron phase fraction below TC upto T^* explained by MGC model can be related to sharp decrease in resistivity. Thus, modified GC model explains well the resistivity behavior near T_c. This model also describes the temperature variation of critical magnetic field quite well. Moreover, for some HTS along with the previous electronic and phononic contribution in specific heat a low temperature Schottky contribution is added because of specific upturn at very low temperatures. However, the contribution is negligible near T_c. Although till now many sophisticated microscopic development has been suggested to describe the effect of electron-phonon interaction in superconductivity, the above modified GC model explains the phenomenological importance of phonon interaction to understand many properties in superconducting phase pretty well.

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