Spin Hall Conductance In a Two-Dimensional Tight-Binding Model In The Presence Of Rashba Spin-Orbit Interaction And Random Impurities

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Abstract

The spin-Hall conductance in a two-dimensional tight-binding model is studied in the presence of Rashba spin-orbit interaction and random impurities. The conventional definition of spin current falls through for a system with Rashba spin-orbit interaction because of the non-conservation of the spin magnetic moment in the presence of Rashba coupling. Using a modified definition of spin-current operator, the relaxation time for electron-impurity scattering is calculated with the help of Matsubara Green function and the spin-Hall conductivity is determined by employing the Kubo-Greenwood formalism. Our results show the explicit behavior of the spin-Hall conductance as a function of chemical potential, Rashba spin-orbit coupling constant and the impurity strength

Keywords: *Rashba Spin-Orbit Interaction, Random Impurities, Kubo Greenwood formalism, Spin Hall Conductivity, Relaxation Time*

1. Introduction

Recent years have witnessed a flurry of investigations on the subject of spin transport in low-dimensional systems. In this connection, several new ideas have been mooted for the realization of new devices that would mainly rely on the spin degrees of freedom of the electrons [1,3]. Of late, both intrinsic and extrinsic spin-Hall effect [5-8] have attracted considerable attention. However majority of the works reported so far have used the conventional definition of spin-current$\left(\frac{1}{2}[vs^z + s^z v]\right)$, which unfortunately falls through in the presence of Rashba spin-orbit interaction (RSOI) because the spin magnetic moment of the system is not conserved in the presence of the Rashba coupling. Furthermore, the conventional definition of spin current gives a finite result in insulators even with localized states. To get rid of the afore-mentioned difficulties, we have constructed following the definition suggested by Shi et al. [1] a modified spin-current operator for our two-dimensional tight-binding model with spin-orbit coupling in the presence of impurity.

We consider a two-dimensional electron system with RSOI in the presence of impurity within the framework of the tight-binding model. The Hamiltonian for the present system can be written as

$$H = H_0 + H_{int} \quad , \tag{1}$$

with

$$H_0 = \varepsilon_0 \sum_i c_i^\dagger c_i + t \sum_{<ij>} (c_i^\dagger c_j + h.c) - i\alpha_R \sum_{<i,j>} \left(c_{ix,iy}^\dagger \sigma^y c_{ix+1,iy} + h.c\right)$$

$$+ i\alpha_R \sum_{<i,j>} (c_{ix,iy}^\dagger \sigma^x c_{ix,iy+1} + h.c), \tag{2}$$

$$H_{int} = \sum \epsilon_i c_i^\dagger c_i = \sum_{i,l} v\delta(r_i - r_l) c_i^\dagger c_i \ , \tag{3}$$

where$\varepsilon_0$is the site energy, t is the hopping integral, $\alpha_R$is the Rashba spin-orbit coupling constant, v is the impurity potential strength, $(\sigma_x , \sigma_y , \sigma_z)$ are the Pauli matrices and

$$c_i = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} , \quad c_i^\dagger = \begin{pmatrix} c_{i\uparrow}^\dagger & c_{i\downarrow}^\dagger \end{pmatrix} \ . \tag{4}$$

$c_{i\uparrow/\downarrow}^\dagger \left(c_{i\uparrow/\downarrow}\right)$is the creation (annihilation) operator for a spin-up/spin-down electron at the i-th site. In the presence of Rashba spin-orbit interaction, the spin degeneracy is lifted leading to two non-degenerate bands. When the impurity is present, the charge carriers will acquire a finite relaxation time τ through elastic scattering. This relaxation time can be calculated from the imaginary part of the self-energy.

2. Theoretical Formalism

2.1. Spin and Charge currents

We define the spin polarization operator as

$$\vec{\mathrm{P}}^{s_z} = \sum_{x_i ,y_i} \vec{R}_{x_i,y_i} c_{x_i,y_i}^\dagger \sigma^z \ , \tag{5}$$

so that the spin current can be written as

$$J^{s_z} = \frac{\partial \vec{\mathrm{P}}^{s_z}}{\partial t} = i \left[H , \sum_{x_i ,y_i} \vec{R}_{x_i,y_i} c_{x_i,y_i}^\dagger \sigma^z c_{x_i,y_i} \right] \ . \tag{6}$$

Similarly, the charge polarization and the charge current can be written as

$$\vec{\mathrm{P}}^{c} = \sum_{x_i ,y_i} \vec{R}_{x_i,y_i} c_{x_i,y_i}^\dagger I c_{x_i,y_i} \ , \tag{7}$$

$$J^{c} = \frac{\partial \vec{\mathrm{P}}^{c}}{\partial t} = i \left[H , \sum_{x_i ,y_i} \vec{R}_{x_i,y_i} c_{x_i,y_i}^\dagger I c_{x_i,y_i} \right] \ . \tag{8}$$

2.2 Spin Hall Conductivity

According to the Kubo formalism, the spin-Hall conductivity is given by

$$\sigma_{xy}^{s_z} = -\frac{e\hbar}{\pi}\sum_{M\neq N}\frac{Im\left[< M\left|J_x^{s_z}\right|N >< N\left|J_y^c\right|M >\right]}{(E_M - E_N)^2 + \left(\frac{\hbar}{\tau(\mu)}\right)^2}\left(f_F(E_M) - f_F(E_n)\right), \quad (9)$$

where $|M >, |N >$ are the eigen states of H_0 belonging to the eigenvalues E_M and E_N respectively and $\tau(\mu)$ is the configuration-averaged relaxation time of the electron. The spin-Hall conductivity is calculated by substituting Eqs. (6) and (8) in Eq. (9)

2. Numerical results

We first calculate the spin and charge currents and the configuration-average relaxation times for the up-spin and down-spin electrons using the Mastubara Green functions. The relaxation times for the up and down-spin electrons turn out to be equal. The spin-Hall conductivity is finally obtained by using the Kubo-Greenwood formula. Fig (1) shows the variation of spin-Hall conductivity (SHE) as a function of α_R for different values of v. Below a certain critical value of α_R (say α_{RC}) (which depends on v), SHC remanis vanishingly snall and as α_R exceeds α_{RC}, SHE increases with α_R motonically and quite rapidly.Fig. 2 presents the behaviour of SHC as a function of v for different values of μ. As expected, SHC decreases with increasing v and becomes zero above a certain value of v. The figure also shows that at small v, SHC increases with the chemical potential.

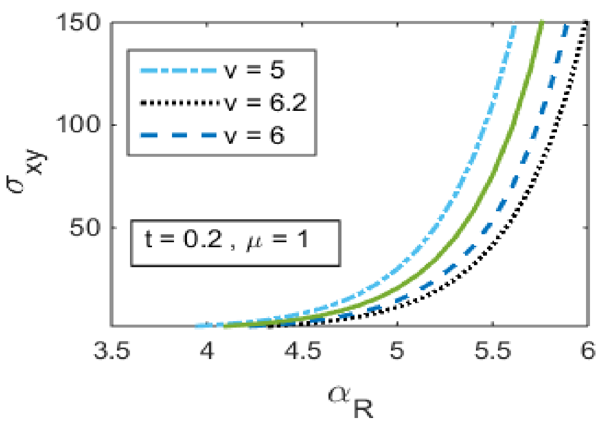

Fig.1 Spin Hall Conductance vs RSOI strength

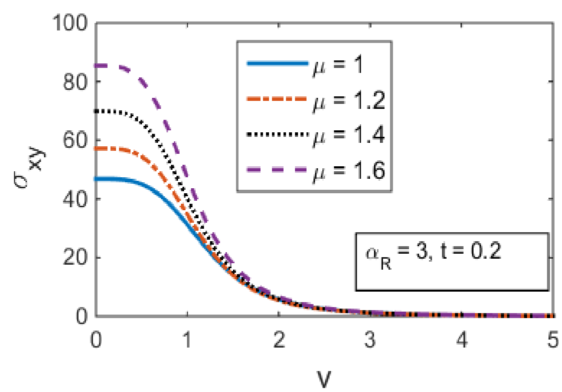

Fig.2 Spin Hall Conductance vs Impurity

4. Conclusion

Using a modified definition of spin-current operator, SHC is calculated using the Matsubara Green function technique and the Kubo-Greenwood formula. It is shown that below a critical value of α_R, SHC is zero and above this critical α_R, SHC increases monotonically and rapidly. As a function of the impurity strength v, SHC, as expected, decreases quite rapidly with increasing v. At small values of v, SHC turns out to be larger for smaller values of the chemical potential.

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