Some Studies on K-essence Lagrangian

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Abstract

K-essence Lagrangian is studied in the context of an early universe epoch when t→ 0. Equation of state parameter $\omega < -1$ as well as deceleration parameter $q_0 < -1$, indicates an accelerated expansion of the universe at an early epoch of time driven by negative pressure generated by dark energy.

Keywords: K−*essence , Lagrangian, Dark energy, Equation of state, Deceleration parameter.*

1. Introduction

From the observation of Type 1a Supernovae by The Supernova Cosmology Project [1-4] and the High-Z-Supernova search team [5,6] it was first established that the universe is undergoing accelerated expansion. Recent observations of luminosity distance of the Type 1a Supernovae (Sne 1a) and other independent observations like Cosmic Microwave Background anisotropies measured with WMAP satellite [7] and Planck satellite [8], Baryon Acoustic Oscillations (BAO) [9] and measurement of oscillations present in the matter power spectrum through large scale surveys [10] indicates that there is some exotic component with negative pressure, known as dark energy, which constitutes about 70% component of the universe and is responsible for the expansion of the universe.

Different cosmological model like -λ CDM model, Chaplygin gas model, Quintessence model and k-essence model has been established to understand the role of dark energy in the universe. In this paper, we have chosen k-essence model in an early epoch of time as our field of study.

The idea of k-essence scalar field $\phi(\mathrm{r},\mathrm{t})$having a non-canonical kinetic term defined as$\mathrm{X} = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$, first came in models of inflation [11, 12]. For such scalar fields kinetic energy dominates over the potential energy, hence the nomenclature of k-essence. Subsequently k-essence fields were shown to lead to models of dark energy [13-17]. The action for the k-essence scalar field $\phi(\mathrm{r},\mathrm{t})$ minimally coupled to background spacetime metric $g_{\mu\nu}$ is given by [15-17]

$$S_k[\phi, g_{\mu\nu}] = \int d^4 x \sqrt{-g} L(\phi, \mathrm{X}) \qquad (1)$$

where, $L(\phi, \mathrm{X})$ is the k-essence Lagrangian.

We will consider the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric of the form

$$\mathrm{dS}^2 = \mathrm{dt}^2 - a^2(t)((dx^1)^2 + (dx^2)^2 + (dx^3)^2) \quad (2)$$

where, $a(t)$ is the cosmological scale factor.

The Lagrangian (pressure) [18-20] for the k-essence field is given by

$$L[\phi, \mathrm{X}] = \mathrm{p} = \mathrm{V}(\phi)F(X) \quad (3)$$

where, $V(\phi)$ is the scalar field potential and $F(X)$is the kinetic part.

Comparing energy-momentum tensor of perfect fluid

$$T_{\mu\nu} = (\rho + \mathrm{p})U_\mu U_\nu - \mathrm{p}g_{\mu\nu} \quad (4)$$

and the energy -momentum tensor with respect to action for k-essence scalar field

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_k}{\delta g^{\mu\nu}} = L_X \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu}L \quad (5)$$

where, $L_X = \frac{\mathrm{dL}}{\mathrm{dX}}$, we get the energy density for the k-essence field as

$$\rho = \mathrm{V}(\phi)[2\mathrm{X}\mathrm{F}_X - F(X)] \quad (6)$$

where, $F_X = \frac{\mathrm{dF}}{\mathrm{dX}}$.

2. Scaling relation

Equation of continuity for the perfect fluid is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (7)$$

where, H is the Hubble parameter defined as $\mathrm{H} = \frac{\dot{a}}{a}$.

From (6) we get

$$\dot{\rho} = V(\phi)[F_X + 2XF_{XX}]\dot{X} + \dot{\phi}[2XF_X - F(X)]V_\phi \quad (8)$$

where, $F_X = \frac{\mathrm{dF}}{\mathrm{dX}}$, $F_{\mathrm{XX}} = \frac{\mathrm{dF}_x}{\mathrm{dX}}$ and $V_\varphi = \frac{\mathrm{dV}}{\mathrm{d}\varphi}$.

Putting (3), (6) and (8) in equation of continuity (7) , yields

$$V(\varphi)[F_X + 2\mathrm{XF}_{\mathrm{XX}}]\dot{X} + \dot{\phi}[2\mathrm{XF}_X - F(X)]V_\varphi + 6\mathrm{HV}(\varphi)\mathrm{XF}_X = 0 \quad (9)$$

For a constant potential, $V_\phi = 0$, so that equation (9) becomes

$$[F_X + 2\mathrm{XF}_{\mathrm{XX}}]\frac{\mathrm{dX}}{\mathrm{dt}} + 6\mathrm{HXF}_X = 0 \quad (10)$$

Since $\mathrm{H} = \frac{1}{a}\frac{\mathrm{da}}{\mathrm{dt}}$, this reduces to the form

$$\left[\frac{1}{X} + 2\frac{F_{XX}}{F_X}\right] \mathrm{dX} = -6\frac{\mathrm{da}}{a} \qquad (11)$$

On first integration we get

$$\sqrt{X}F_X = \mathrm{Ca}^{-3} \qquad (12)$$

where, C is an integration constant. This relates scale factor with the kinetic energy part, hence equation (12) is known as the scaling relation [20,21] and plays a very important role in developing k-essence Lagrangian [21-24].

3. K-essence Lagrangian

First Friedmann equation relates the energy density with Hubble parameter and is given by

$$H^2 = \frac{8\pi G}{3}\rho \qquad (13)$$

Putting (6) in (13) we get

$$H^2 = \frac{8\pi G}{3}V[F(X) - 2\mathrm{XF}_X] \qquad (14)$$

Since we have considered constant potential hence we will write $V(\phi) = \mathrm{V} = \mathrm{constant}.$

From (12) we get $F_X = \frac{\mathrm{Ca}^{-3}}{\sqrt{X}}$, substituting in (14) we get

$$F(X) = \frac{3}{8\pi GV}H^2 + 2C\sqrt{X}a^{-3} \qquad (15)$$

Let $\mathrm{q} = \ln a$, so that Hubble parameter becomes $\mathrm{H} = \frac{\dot{a}}{a} = \dot{q}$. Considering present observable universe to be homogeneous , we will consider scalar field to be function of time only i.e., $\phi(\mathrm{r,t}) = \phi(t)$,so that $\mathrm{X} = \frac{1}{2}\dot{\phi}^2$. Thus equation (15) becomes

$$F(X) = \frac{3}{8\pi GV}\dot{q}^2 + \mathrm{C}\sqrt{2}\dot{\phi}e^{-3q} \qquad (16)$$

Putting (16) in (3) we get

$$\mathrm{L} = -c_1\dot{q}^2 - c_2\dot{\phi}e^{-3q} \qquad (17)$$

where, $c_1 = \frac{3}{8\pi G}$ and $c_2 = \mathrm{C}\sqrt{2}V$.

This is the k-essence Lagrangian in canonical form with first term as the kinetic term and second term as an interaction term.

Interesting fact is logarithmic of the scale factor plays the role of a dynamical kinetic term in this Lagrangian. Setting up of Euler Lagrangian equation and solution has been studied in Ref [21].

In this work we will consider an early epoch condition of the universe and will try to develop k-essence theory to understand the nature of dark energy at the beginning of the universe.

At an early epoch Scale factor a(t) is very small so that q is also very small. Expanding 2nd term of (13) up to q^2 term we get

$$K = -c_1\dot{q}^2 - c_2\dot{\phi}\left(1 - 3q + \frac{9}{2}q^2\right) \qquad (18)$$

Scaling $q \rightarrow q + \frac{1}{3}$, equation (18) becomes

$$K = -c_1\dot{q}^2 - \frac{9}{2}c_2\dot{\phi}q^2 - \frac{1}{2}\dot{\phi} \qquad (19)$$

This Lagrangian with some modification has been studied in context to Ermakov invariant as an early probe to understand the universe in Ref [22]. Here in this work we will try to determine two most important cosmological parameters, equation of state parameter $(\omega)$and deceleration parameter (q_0) to understand the role of dark energy at an early epoch of time.

4. Equation of state parameter ω

Since Lagrangian is equivalent to pressure, hence we can write the pressure (p) of k-essence field at a very early epoch when scale factor is very small as

$$p = -c_1\dot{q}^2 - \frac{9}{2}c_2\dot{\phi}q^2 - \frac{1}{2}\dot{\phi} \qquad (20)$$

The energy density is given by the Friedmann equation

$$\rho = \frac{3}{8\pi G}H^2 = c_1\dot{q}^2 \qquad (21)$$

Equation of state parameter is defined as $\omega = \frac{p}{\rho}$. From (20) and (21) we get

$$\omega = -1 - \frac{\frac{9}{2}c_2\dot{\phi}q^2}{c_1\dot{q}^2} - \frac{\frac{1}{2}c_2\dot{\phi}}{c_1\dot{q}^2} \qquad (22)$$

Since $c_{1,}c_{2,}q^2$ and $\dot{q}^2$ are always positive, we see EOS (ω) remains negative if $\dot{\phi}$ remains positive i.e., if $\phi \sim t$. This shows the scalar field is inflationary in nature. Thus equation (18) shows that $\omega < -1$. This clearly indicates the presence of dark energy as a generator of negative pressure at an earlier epoch of time during the inflationary phase of evolution of the universe.

5. Deceleration parameter q_0

One of the important cosmological parameter is deceleration parameter $q_0 = \frac{-a\ddot{a}}{\dot{a}^2}$, which determines the acceleration or deceleration of the expansion of the universe. It is defined in terms of EOS (ω) as,

$$q_0 = \frac{1}{2}(1 + 3\omega) \qquad (23)$$

Equation (22) shows that $\omega < -1$ for an early epoch, hence (23) indicates $q_0 < -1$ as well. Negativity of q_0 shows that we have an accelerated expansion of the universe for $t \to 0$ i.e., at an earlier epoch of time.

6. Conclusion

In this paper we have studied the k-essence dark energy model under the context of an earlier epoch of time during the inflationary stage of evolution of the universe. Equation of state parameter $\omega < -1$ indicates the presence of dark energy as a source of negative pressure. Deceleration parameter $q_0 < -1$ indicates an accelerated expansion of the universe. Thus for an early epoch of time i.e., when, $t \to 0$, both the cosmological parameter ω and q_0 indicates an accelerated expansion of the universe driven by negative pressure identified as dark energy .

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